CALDAM 2023

Indo-Dutch Pre-Conference School on Algorithms and Combinatorics

February 6 - 7, 2023

Young Researchers' Forum

Booklet of Abstracts

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CALDAM 2023 Pre-Conference School on Algorithms and Combinatorics

February 6-7, 2023

Organized by:

Dhirubhai Ambani Institute of
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Coordinated by:

- Bodo Manthey, (Co-Convenor)
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- R. S. Lekshmi,
  PSG College of Technology, India
- Sunitha VadivelMurugan, (Co-Convenor)
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Preface

Discrete Mathematics is viewed as one among the fundamental fields to understand mathematical grounds of Computer Science, quintessentially Algorithms, Cryptography, Graph Theory, Theory of Computing, and various other related disciplines. Although the above-mentioned fields have origins centuries ago, these fields are evergreen and prove to be highly promising due to immense research conducted in numerous universities and research labs around the globe. A wide range of computationally simple to intensive problems can be defined on the premise of a large family of discrete objects, which when solved have enormous applications in plenty of domains.

The Indo-Dutch Pre-Conference School is an initiative to improve research through collaboration with researchers worldwide. The school intends to expose current trends in Algorithms and Combinatorics for PhD students and teachers in Computer Science and Discrete Mathematics. The knowledge dissemination and sharing will encourage inquisitive students to become prospective researchers.

Young Reseachers' Forum (YRF) is a pioneer effort in the series of CALDAM conference. The forum is an opportunity for interested participants to present a problem they are attempting to solve in the theme of the conference. Presentations by students in the forum shall be viewed as an open discussion session and the deliberations with speakers and experts will be an inspiration and motivation to the students to pursue a research career in the fields of Algorithms and Combinatorics. Together with the conference, an interactive session of this kind enables students, researchers and teachers to engage in exchange of knowledge, ideas and research methods that benefits all the participants involved in the Pre-Conference School.

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February 05, 2023

Lecture Schedule

Time	February 06, 2023	February 07, 2023
08:00 – 09:15	Registration & Inauguration	
09:15 – 10:30	Technical Session - 1 **Arrangements, Partitions, and Applications** Mark de Berg	Technical Session - 4 **Smoothed Analysis and its Applications to Local Search Heuristics** Jessie van Rhijn
10:30 – 11:00	**Break**	
	Technical Session - 2 **Graph Coloring Problems** Rishi Ranjan Singh	Technical Session - 5 **Graph Classes arising from the Perfect Matching Polytope** Nishad Kothari
11:45 – 12:30	**Games on Graphs and Eternal Vertex Cover** Neeldhara Misra	**Some Open Problems in Computational Group Theory** Bireswar Das
12:30 – 13:45	**Lunch**	
13:45 – 15:00	Technical Session - 3 **From Approximate to Exact Integer Programming** Daniel Dadush	Technical Session - 6 **Random Metrics in the Analysis of Algorithms** Bodo Manthey
15:00 – 15:30	**Break**	
15:30 – 17:00	Young Researchers Forum **Short Presentations** by Manoj Changat, A. Mohanapriya, Pavan P.D., Supraja D.K., Adri Bhattacharya, Ritam Manna Mitra, Sangam Balachandar Reddy, Saraswati Girish Nanoti, Pavithra R.	Participants' Feedback & Conclusion
19:30 onwards	**Dinner**	

Presentations - Young Researchers' Forum

Problems on Transit Functions on graphs

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7, February 2023

Abstract

In this presentation, the notion of transit functions in graphs is introduced as a generalization of betweenness, intervals, and convexity. In particular, we discuss transit functions defined by various types of paths such as shortest paths, induced paths, any path, paths of length exactly two, etc. in graphs. Problems on Betweenness, intervals, and convexity of these different types of transit functions are addressed.

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Total Outer-Connected Domination on convex split graphs - Complexity Results

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Abstract

For a connected graph G, the total outer-connected dominating set problem (TOCD) asks for a partition of $V(G)$ into D and $V(G) \setminus D$ such that D is a total dominating set of G and $G[V(G) \setminus D]$ is connected. TOCD is NP-complete on general graphs, chordal graphs and split graphs [1]. In this work, we introduce a subclass of split graphs called star-convex split graphs and strengthen the NP-completeness result of split graphs; TOCD is NP-complete on star-convex split graphs with convexity on clique (independent set). In the parameterized setting, it is interesting to observe that the parameterized version of total outer-connected domination problem with respect to the solution size is W[2]-hard on star-convex split graphs with convexity on clique, and is W[1]-hard on star-convex split graphs with convexity on independent set. Further, we obtain an interesting dichotomy for TOCD on star-convex split graphs with convexity on clique; TOCD is polynomial-time solvable if no pendant vertices, and NP-complete, otherwise. If the convexity is on the independent set, then the dichotomy is; on K1,6-free star-convex split graphs with an imaginary star for clique vertices is K1,3, TOCD is NP-complete, and polynomial-time solvable, otherwise. Further, we prove that star-convex split graph with convexity on independent set does not admit $(1-\epsilon)\ln|V(G)|$-approximation algorithm unless $NP \subseteq DTIME(n^{O(\log\log n)})$. Furthermore, we prove that for path-convex split graphs finding a minimum total-outer connected domination is polynomial-time solvable.

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On Deeply Critical Oriented Cliques

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Abstract

In this work we consider arc criticality in colourings of oriented graphs. We study deeply critical oriented graphs, those graphs for which the removal of any arc results in a decrease of the oriented chromatic number by 2. We prove the existence of deeply critical oriented cliques of every odd order $n \geq 9$, closing an open question posed by Borodin et al. [*Journal of Combinatorial Theory, Series B, 81(1):150–155, 2001*]. Additionally, we prove the non-existence of deeply critical oriented cliques among the family of circulant oriented cliques of even order.

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Radio k-labeling of the infinite path

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February 5, 2023

Abstract

The radio k-chromatic number $rc_k(G)$ of a graph G is the minimum integer ℓ such that there is a mapping f from the vertices of G to the set of integers $\{0, 1, \ldots, \ell\}$ satisfying $|f(u) - f(v)| \geq k + 1 - d(u, v)$ for any two distinct vertices $u, v \in V(G)$, where $d(u, v)$ denotes the distance between u and v. To date, the radio k-chromatic number of finite paths [5] and square of finite paths [4] is computed exactly when k is equal to the diameter of the graph. Moreover, the exact value of the radio k-chromatic number of an arbitrary power of a finite path is also computed when the diameter of the path is strictly smaller than k in [1]. The radio k-chromatic number of the infinite path is conjectured by Kchikech et al. 2007 [3]. Close lower and upper bounds for the radio k-chromatic number are given for the infinite path by two different bodies of works [2, 3]. We discuss if the known lower bound can be improved further to coincide with the upper bound for infinite path.

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Treasure Hunt in Graph using Pebbles

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1 Abstract

In this paper, we study the treasure hunt problem in a graph by a mobile agent. The nodes in the graph $G = (V, E)$ are anonymous and the edges incident to a vertex $v \in V$ whose degree is $deg(v)$ and they are labeled arbitrarily as $0, 1, \ldots, deg(v) - 1$. At a node t in G a stationary object, called *treasure* is located. The mobile agent that is initially located at a node s in G, the starting point of the agent, must find the treasure by reaching the node t. The distance from s to t is D. The *time* required to find the treasure is the total number of edges the agent visits before it finds the treasure. The agent neither have any prior knowledge about the graph nor the position of the treasure. An oracle that knows the graph, the agent's initial position, and the position of the treasure, places some pebbles on the nodes, at most one per node, of the graph to guide the agent towards the treasure.

This paper aims to study the trade-off between the number of pebbles provided and the time required to find the treasure. To be specific, we aim to answer the following question:

- "What is the minimum time for treasure hunt in a graph with maximum degree Δ and diameter D if k pebbles are placed?"

We answer the above question when $k < D$ or $k = cD$ for some positive integer c. We design efficient algorithms for the agent for different values of k. We also propose an almost matching lower bound result for $k < D$. In the next section, we discuss some of our contribution.

1.1 Contribution

We study the trade-off between the number of pebbles (k) provided by the oracle and the associated time required to find the treasure. The contributions in this paper are mentioned below.

- For $k < \frac{D}{2}$ pebbles, we propose an algorithm that finds the treasure in a graph at time $O(D\Delta^{\frac{D}{(2\eta+1)}})$, where $\eta = \frac{k}{3}$.

- For $\frac{D}{2} \leq k < D$, we propose a treasure hunt algorithm with time complexity $O(k\Delta^{\frac{D}{k+1}})$.

- In case of bipartite graphs, the proposed algorithm for treasure hunt has time complexity $O(k\Delta^{\frac{D}{k}})$ for $0 < k < D$.

- For $k = cD$ where c is any positive integer, we give an algorithm that finds the treasure in time $O\left[cD(\frac{\Delta}{2^{c/2}})^2 + cD\right]$

- We propose a lower bound result $\Omega((\frac{k}{e})^{\frac{k}{k+1}}(\Delta - 1)^{\frac{D}{k+1}})$ on time of treasure hunt for $0 < k < D$.

On Voronoi Games

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CALDAM 2023

1 Abstract

Competitive facility location is concerned with the favorable placement of facilities by competing market players. In such a scenario, when the users choose the facilities based on the nearest-neighbor rule, the optimization criteria is to maximize the cardinality or the area of the service zone depending on whether the demand region is discrete or continuous, respectively. In a plane, two players P_1 and P_2 place facilities on the plane taking turns for a finite number of rounds. The game may be played for multiple rounds. Users may place multiple facilities in each round. We are interested in algorithms for optimal strategies for placements of facilities by both players. We study different versions of the Voronoi Game and pose some questions regarding existing algorithms.

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Parameterized complexity of $[1, j]$-domination and $[1, j]$-total domination problems

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Abstract. Given a graph $G = (V, E)$, a Dominating set is a set $D \subseteq V$ such that each vertex in $V \setminus D$ is adjacent to at least one vertex in D. A set $D \subseteq V$ is a $[1, j]$-Dominating set if every vertex $v \in V \setminus D$ has at least one and at most j neighbours in D. A set $D \subseteq V$ is a $[1, j]$-Total dominating set if every vertex $v \in V$ has at least one and at most j neighbours in D.

Bishnu et al. [2] proved that $[1, j]$-Dominating set problem is NP-hard even for chordal and planar graphs. $[1, 2]$-Total dominating set is NP-complete even for bipartite graphs [4]. $[1, j]$-Dominating set and $[1, j]$-Total dominating set can be solved in time $O^*(j + 2)^{tw}$ and $O^*(2j+2)^{tw}$ respectively, on graphs of treewidth at most tw [3]. Recently, Meybodi et al. [1] proved that $[1, j]$-Dominating set parameterized by the solution size is W[1]-hard on d-degenerate graphs. They have also shown that $[1, 2]$-Total dominating set cannot be solved in time $O^*(4 - \epsilon)^{tw}$ (unless SETH fails).

In this talk, we would like to discuss the following couple of interesting open problems from the literature:

1. Parameterized complexity of $[1, j]$-Dominating set problem when the degeneracy is smaller or equal to j parameterized by the solution size.
2. The lower bound for $[1, 2]$-Dominating set problem, or the lower bound for $[1, j]$-Dominating set problem in general.

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Eternal Vertex Cover

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Abstract

The ETERNAL VERTEX COVER problem is a dynamic variant of the vertex cover problem. We have a two player game in which guards are placed on some vertices of a graph. In every move, one player (the attacker) attacks an edge. In response to the attack, the second player (the defender) moves some of the guards along the edges of the graph in such a manner that at least one guard moves along the attacked edge. If such a movement is not possible, then the attacker wins. If the defender can defend the graph against an infinite sequence of attacks, then the defender wins.

The minimum number of guards with which the defender has a winning strategy is called the eternal vertex cover number of the graph G.

It is clear that $evc(G)$ is *at least* $mvc(G)$, the size of a minimum vertex cover of G. We say that G is *Spartan* if $evc(G) = mvc(G)$. The characterization of Spartan graphs has been largely open. In the setting of bipartite graphs on $2n$ vertices where every edge belongs to a perfect matching, an easy strategy is to have n guards that always move along perfect matchings in response to attacks. We show that these are essentially the only Spartan bipartite graphs.

On general graphs, the computational problem of determining the minimum eternal vertex cover number is NP-hard and admits a 2-approximation algorithm and an exponential kernel. We show that Eternal Vertex Cover is NP-hard and does not admit a polynomial compression even on bipartite graphs of diameter six.

2012 ACM Subject Classification Theory of computation → Parameterized complexity and exact algorithms; Theory of computation → Algorithm design techniques

Keywords and phrases eternal vertex cover, perfect matchings, bipartite graphs

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On Compressed Zero-divisor Graphs of Finite Commutative Rings

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Abstract

Let R be a commutative ring with $1 \neq 0$. The relation $\sim$ on R defined by $a \sim b$ if and only if $ann_R(a) = ann_R(b)$ is an equivalence relation. The compressed zero-divisor graph, $\Gamma_E(R)$ is the graph associated to R whose vertices are the classes of elements in R except $[0]$ and $[1]$, and with each pair of distinct classes $[x], [y]$ joined by an edge if and only if $[x][y] = 0$. We have studied some properties of $\Gamma_E(R)$, where R is a finite commutative ring with $1 \neq 0$. If $R \cong R_1 \times R_2 \times \cdots \times R_m$ is the local ring decomposition of R and the maximal ideals of R_i are principal ideals, then $\delta(\Gamma_E(R)) = 1$ and the domination number is m. We found an example that shows the graph $\Gamma_E(R)$ may not have a vertex of degree one if the maximal ideal of the local ring R is not a principal ideal. If R is a finite commutative ring with $1 \neq 0$, what would be the domination number of $\Gamma_E(R)$?

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